(Why) Is Helicity Lorentz-Invariant?

Inevitable Features of Long-Range Forces

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arXiv:1302.1198,
arXiv:1302.1577,
arXiv:1302.3225,
arXiv:1305.xxxx,

with Philip Schuster

Massless particle states with momentum k^μ :

$$\underbrace{\vec{\mathbf{J}}.\hat{k}}_{\mathbf{R}}|k,h\rangle = h|k,h\rangle$$

helicity eigenstate

How do Lorentz-transformations affect helicity eigenstates?

Simplest: look at $\Lambda k = k$.

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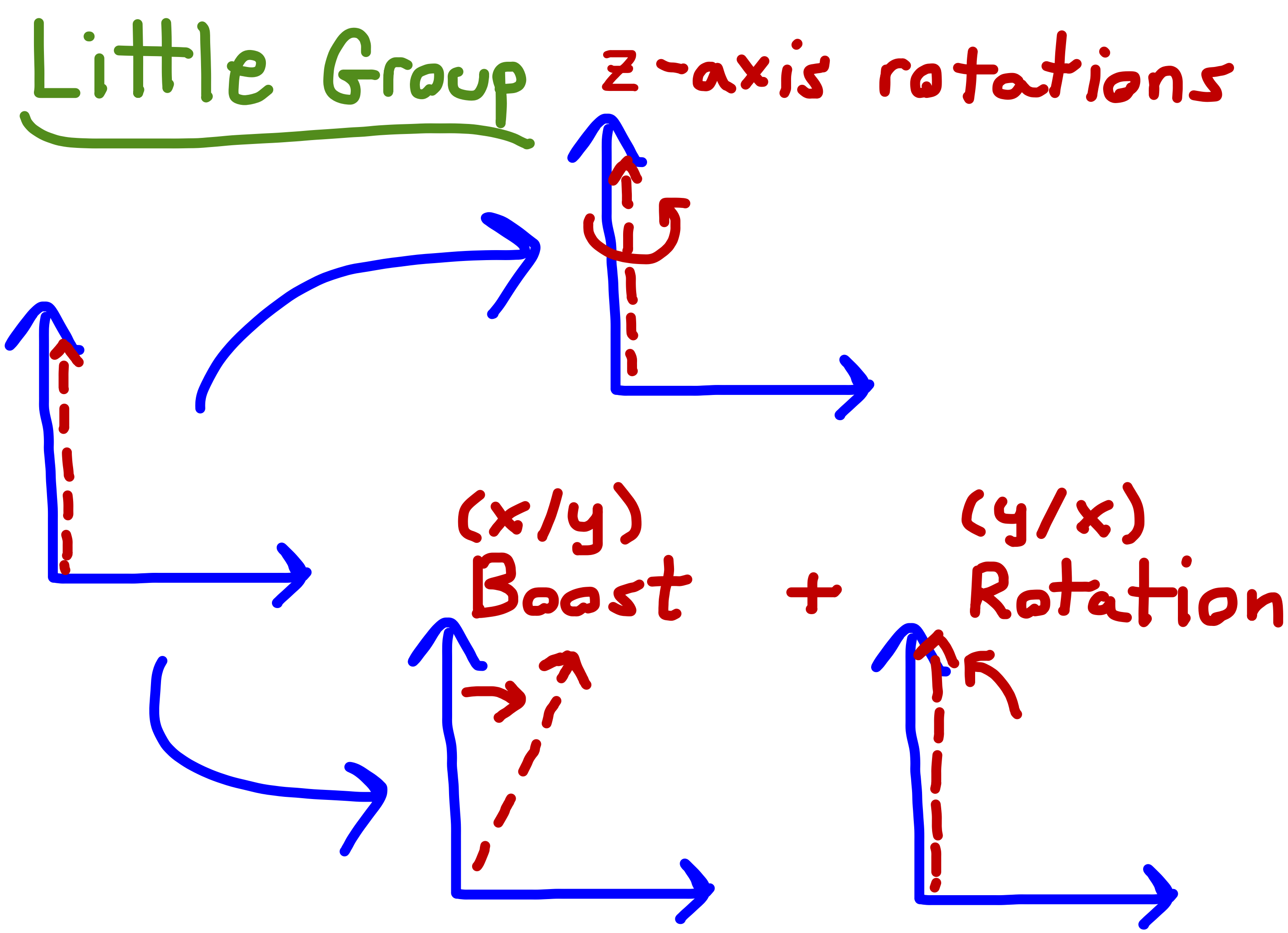
- rotation about $\vec{k}$ axis (generated by $\mathbf{R}$)
- transverse rotation+boost, generated by

$$\mathbf{T}_{1,2} \equiv \vec{\epsilon}_{1,2}.(\vec{\mathbf{K}} \times \vec{k} + \vec{\mathbf{J}} k^0)$$



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- transverse rotation+boost, generated by

$$\mathbf{T}_{1,2} \equiv \vec{\epsilon}_{1,2}.(\vec{\mathbf{K}} \times \vec{k} + \vec{\mathbf{J}} k^0)$$

Action of rotations on states is simple – we ***defined*** $|k, h\rangle$ to be $\mathbf{R}$-eigenstates:

$$e^{i\theta\mathbf{R}}|k, h\rangle = e^{i\theta h}|k, h\rangle$$

...what about $\mathbf{T}$'s?

Simplest: look at $\Lambda k = k$.

- rotation about $\vec{k}$ axis (generated by $\mathbf{R}$)
- transverse rotation+boost, generated by

$$\boxed{\mathbf{T}_{1,2} \equiv \vec{\epsilon}_{1,2}.(\vec{\mathbf{K}} \times \vec{k} + \vec{\mathbf{J}} k^0)}$$

Combinations $\mathbf{T}_\pm \equiv \mathbf{T}_1 \pm i\mathbf{T}_2$ raise and lower helicity by one unit:

$$[\mathbf{R}, \mathbf{T}_\pm] = \pm \mathbf{T}_\pm \qquad [\mathbf{T}_+, \mathbf{T}_-] = 0$$

$$\mathbf{T}_\pm |k, h\rangle = \rho |k, h \pm 1\rangle$$

units of ***momentum***

$$\mathbf{T}_\pm|k,h\rangle = \rho|k,h\pm 1\rangle$$

$$\Rightarrow \boxed{\begin{gathered} e^{ib_a\mathbf{T}_a}|k,h\rangle = \sum_{h'} D_{hh'}(b)|k,h'\rangle \\ D_{hh'}(b) \sim J_{h-h'}(\rho|b|) \end{gathered}}$$

Mix under Lorentz! (like massive polarizations)
unless we enforce ρ=0

Wigner's "continuous-spin" representations

More covariantly:

$$\mathbf{W}^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\mathbf{J}_{\nu\rho}\mathbf{P}_\sigma$$

(components of $\mathbf{W} \propto \mathbf{T}_1$, $\mathbf{T}_2$, and $\mathbf{R}$)

$$\boxed{\mathbf{W}^2|k,h\rangle_\rho = -\rho^2|k,h\rangle_\rho}$$

ρ≠0: all integer *h*'s (or half-integer) present in same representation

massive spin-*s*: $\mathbf{W}^2|p,m\rangle = -m^2 S(S+1)|p,m\rangle$

ρ=0 helicity *h*: $(\mathbf{W}^\mu - h\mathbf{P}^\mu)|k,h\rangle = 0$

What features of long-range forces are inevitable consequences of Poincare+Unitarity?

Massless bosons

h=0

h=1

h=2

h=3

h=4

⋮

Much is known about the $\rho=0$ branch.
What about $\rho\neq 0$?

What features of long-range forces are inevitable consequences of Poincare+Unitarity?

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Massless bosons

~~h=0~~ not radiatively stable

h=1
h=2
coupled to conserved charges

~~h=3~~
~~h=4~~
~~⋮~~
no consistent couplings strong enough to mediate 1/r forces (Weinberg 1960s)

Much is known about the $\rho=0$ branch.
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Massless bosons

h=0

h=1

h=2

If CSPs ***can*** mediate long-range forces, how do they behave?

How does the $\rho \rightarrow 0$ limit work?

What is the $\rho \neq 0$ analogue of the restriction to h=0, 1, 2?

Are $\rho \neq 0$ theories physically viable? How can they be tested?

Outline

1. Physical picture & Motivation for $\rho \neq 0$
   "Continuous-spin" particles (CSPs)

2. Evidence for (tree) interactions and their consequences

3. Gauge Field Theory (an intro)

4. Conclusions

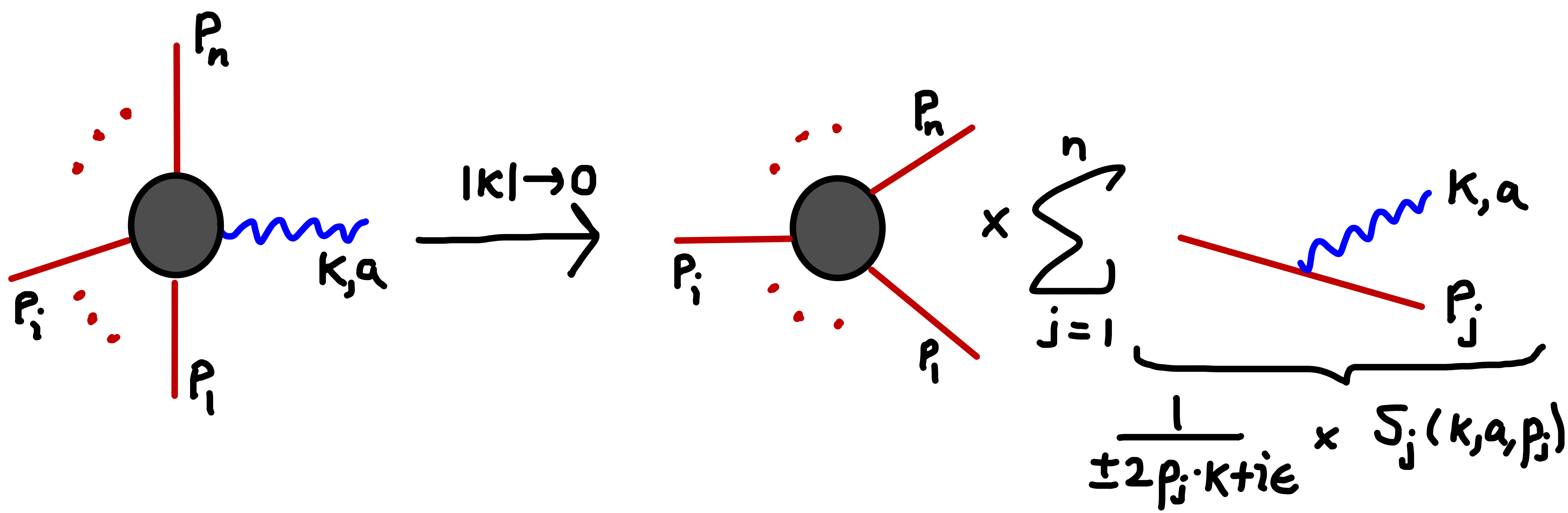
Amplitudes and Their Implications

Lorentz + Unitarity fix *single-CSP emission amplitudes* almost uniquely

- Correspondence with standard helicity amplitudes when $E_{CSP} \gg \rho v$

- Allows viable *approximate* thermodynamics

High-Helicity Soft Limits



must transform like $|k, a\rangle_\rho$

Weinberg "Soft Theorems":
For $h>2$, no Lorentz-covariant solution

Are there analogous constraints on CSPs ?

Single-CSP states: Lorentz Transformations

Helicity/spin basis

$$\vdots$$
$$|3\rangle$$
$$|2\rangle$$
$$|1\rangle$$
$$|0\rangle$$
$$|-1\rangle$$
$$|-2\rangle$$
$$|-3\rangle$$
$$\vdots$$

$$|h\rangle \to e^{ih\theta}|h\rangle$$

rotation eigenstate

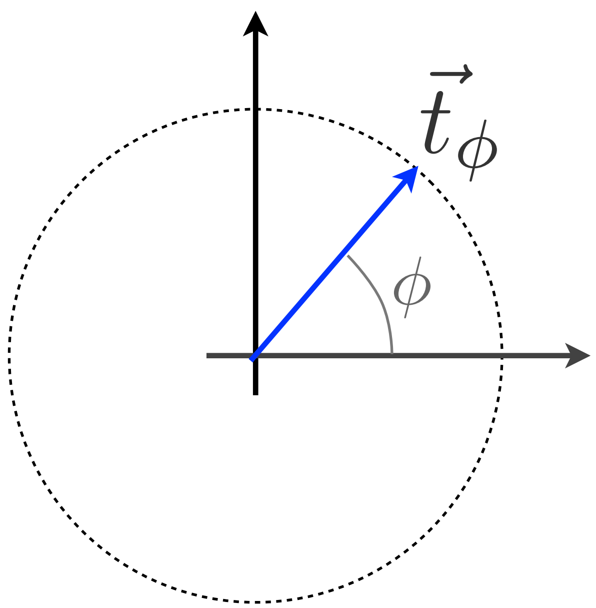
$$|h\rangle \to J_{h-h'}(\rho|\vec{b}|)|h'\rangle$$

translations mix states

$$\langle h|h'\rangle = \delta_{hh'}$$

Single-CSP states: Lorentz Transformations

"Angle" basis $|\phi\rangle \equiv \sum_h e^{ih\phi}|h\rangle$ =E_2 plane-wave



$|\vec{t}_\phi|^2 = \rho^2$

$|\phi\rangle \rightarrow |\phi + \theta\rangle$ rotations mix states

$|\phi\rangle \rightarrow e^{i\vec{b}.\vec{t}_\phi}|\phi\rangle$ **translation eigenstate**

$$\langle\phi|\phi'\rangle = \delta(\phi - \phi')$$

Single-CSP states: Lorentz Transformations

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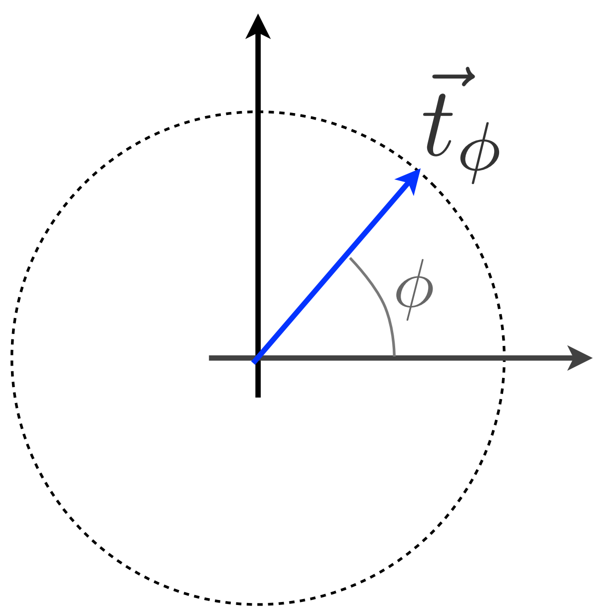


$|\vec{t}_\phi|^2 = \rho^2$

$$|\phi\rangle \to |\phi+\theta\rangle$$

rotations mix states

$$|\phi\rangle \to e^{i\vec{b}.\vec{t}_\phi}|\phi\rangle$$

translation eigenstate

Covariantly: $|k,\epsilon\rangle$ with $\epsilon.k = 0, \epsilon^2 = -1$

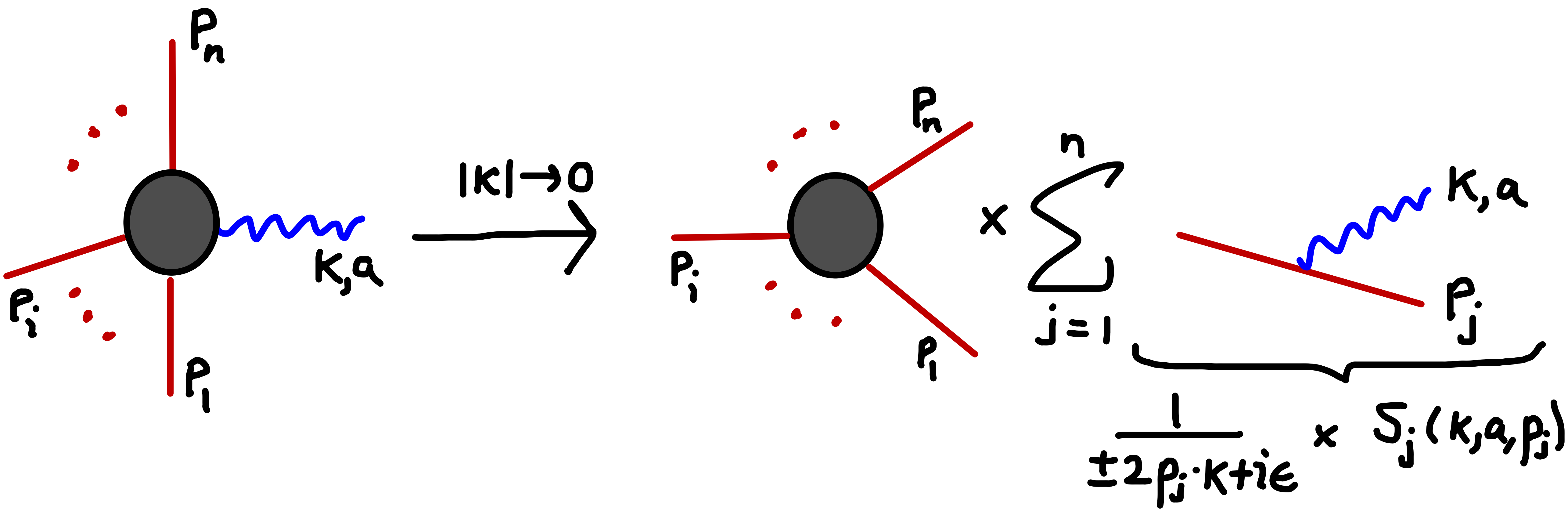
– equivalence $|k,\epsilon+\alpha k\rangle \simeq e^{i\rho\alpha}|k,\epsilon\rangle$

– basis $|k,\epsilon_c\rangle$ with $\epsilon_c^0 = 0 \leftrightarrow |k,\phi\rangle$

(define $\epsilon_c(k,\phi)$)

– simple Lorentz action $|k,\epsilon\rangle \to |\Lambda k, \Lambda\epsilon\rangle$

Single-CSP emission in the soft limit



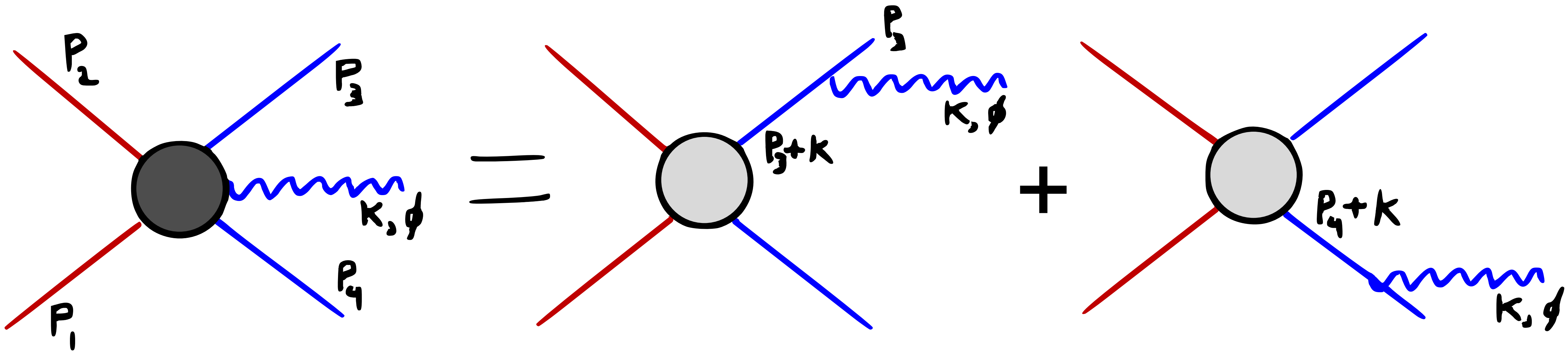
must transform like $|k, a\rangle_\rho$

$$s(\{k, \epsilon + \alpha k\}, p_i) = e^{i\rho\alpha} s(\{k, \epsilon\}, p_i)$$

$$\Rightarrow \boxed{s(\{k, \epsilon\}, p_i) = f(k.p_i) e^{i\rho \frac{p_i \cdot \epsilon}{p_i \cdot k}}}$$

[Schuster & NT 1302.1198]

A simple tree amplitude:

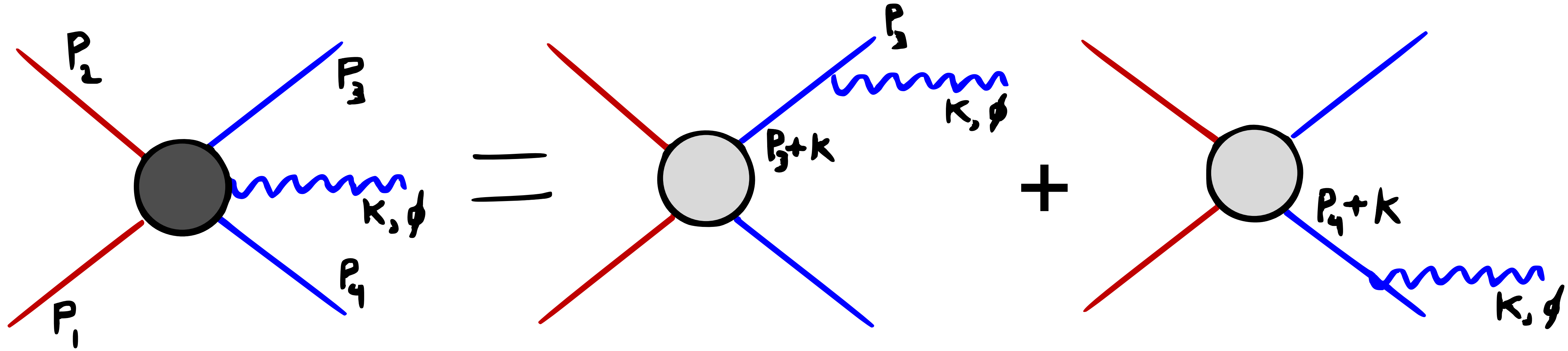


$$A_4 \cdot \frac{1}{(p_3+k)^2+i\epsilon} \cdot s(\{k,\phi\},p_3) + A_4 \cdot \frac{1}{(p_4+k)^2+i\epsilon} \cdot s(\{k,\phi\},p_4)$$

where $s(\{k,\phi\},p_i) = f(k.p_i)e^{i\rho\frac{p_i.\epsilon_C(k\phi)}{p_i.k}}$

($f \rightarrow$ constant a_i for most of this talk, or monomial)

Only phase is ϕ-dependent$\Rightarrow \int \frac{d\phi}{2\pi}|\mathcal{A}|^2$ is finite!

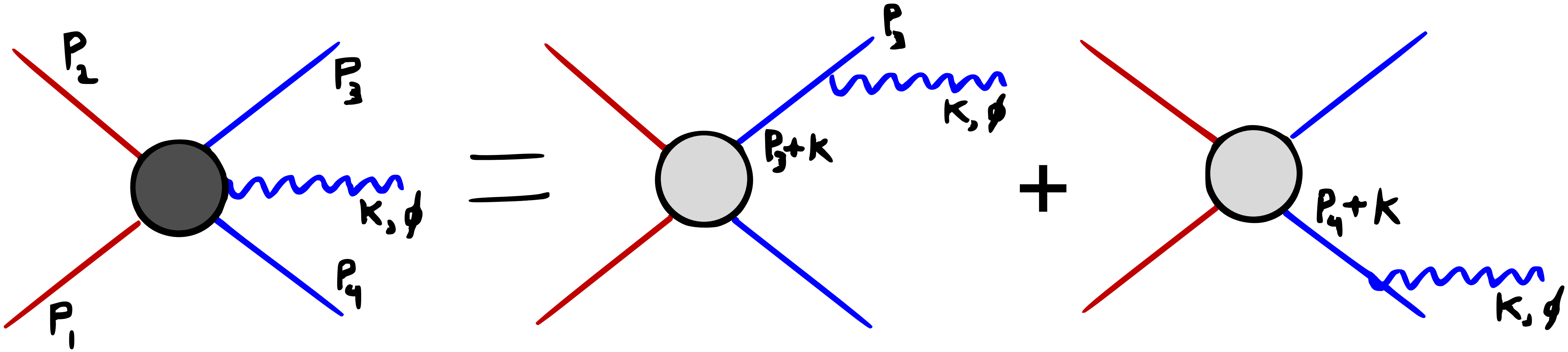


$$A_4 \cdot \frac{1}{(p_3+k)^2+i\epsilon} \cdot s(\{k,\phi\}, p_3) + A_4 \cdot \frac{1}{(p_4+k)^2+i\epsilon} \cdot s(\{k,\phi\}, p_4)$$

$$\int \frac{d\phi}{2\pi} |A(12 \to 34\{k,\phi\})|^2 = |\lambda|^2 \left| \frac{s(\{k,\phi\},p_3)}{(p_3+k)^2+i\epsilon} + \frac{s(\{k,\phi\},p_4)}{(p_4+k)^2+i\epsilon} \right|^2$$

$$= |\lambda|^2 \left(\frac{|a_3|^2}{((p_3+k)^2)^2} + \frac{|a_4|^2}{((p_4+k)^2)^2} + \frac{2Re[a_3 a_4^*] J_0(\rho|z_i - z_j|)}{(p_3+k)^2 (p_4+k)^2} \right)$$

Singularities (only) at on-shell particle poles



$$A_4 \cdot \frac{1}{(p_3+k)^2+i\epsilon} \cdot s(\{k,\phi\}, p_3)$$

$$+ \quad A_4 \cdot \frac{1}{(p_4+k)^2+i\epsilon} \cdot s(\{k,\phi\}, p_4)$$

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ρz = <u>correspondence parameter</u>

(recover scalar result when $\rho z \to 0$)

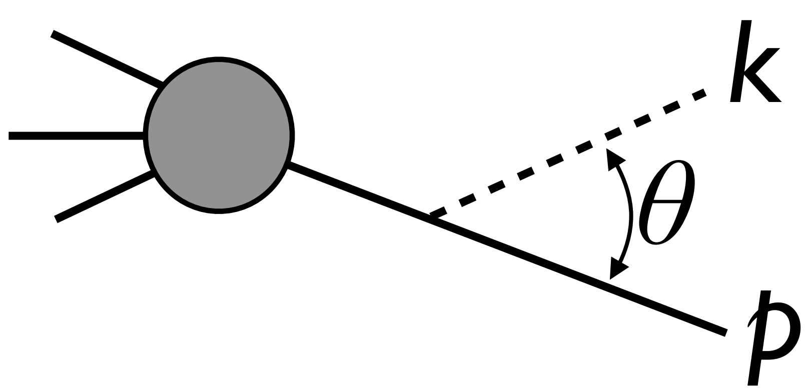
Complex correspondence parameter z_i:

$$z_i \equiv \epsilon_-^c(k).p_i/k.p_i$$

reminiscent of Klein-Nishina, etc.

(ϵ_-^c = *circular* polarization w/ ϵ^0=0, $\epsilon.k$=0)

$$|z| \approx \frac{|\mathbf{p}|\sin\theta}{|\mathbf{k}|(p^0 - |\mathbf{p}|\cos\theta)}$$



For θ~1, $|z| \sim v/|\mathbf{k}|$ so $\rho z \ll 1$ is the limit of high-energy radiation and/or non-relativistic emitters.

Lorentz-invariant quantities depend only on $|z_i – z_j|$

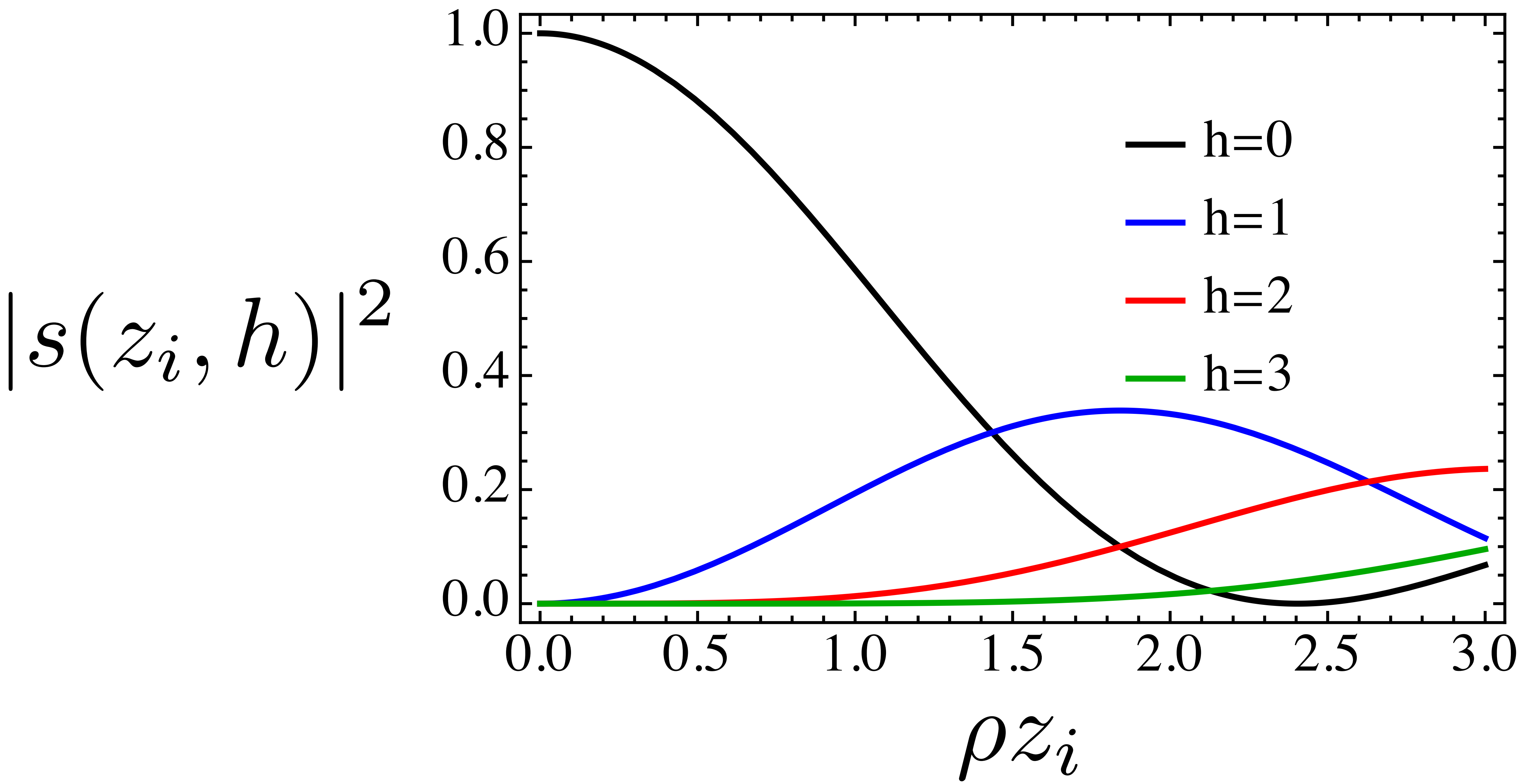
Soft factors are simple in terms of z:

$$s(\{k,\phi\},p_i) = e^{i\rho \mathrm{Re}[e^{i\phi} z_i]} \xrightarrow{\text{Fourier}}$$

$$\begin{aligned} s(\{k,h\},p_i) &= J_h(\rho|z_i|)e^{-ih\arg(\mathrm{z})} \\ &\equiv \tilde{J}_h(\rho z_i) \end{aligned}$$

Lorentz-invariant quantities depend only on $|z_i - z_j|$

Leading behavior at small z_i (=high energy)



Suppression follows from Taylor expansion of J_h

$$J_h(x) \approx \frac{x^h}{2^h h!}(1 - O(x^2) + \dots)$$

For minimal (f=$const$) soft factor and momenta $\gg \rho$,

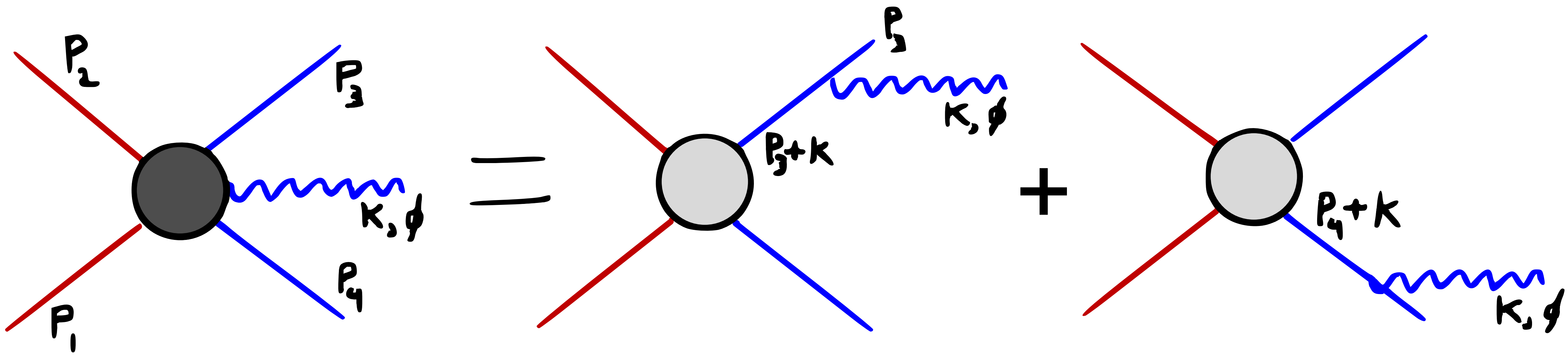
$$A(\{k, h=0\}, p\ldots) = A_{scalar}(1 - \mathcal{O}(\rho z)^2)$$

The h≠0 amplitudes are hierarchically smaller:

$$A(\{k, h=\pm n\}, p...) \sim A_{scalar}(\rho z)^n/n! + ...$$

Helicity correspondence! [1302.3225 Schuster & NT]

More general interactions?



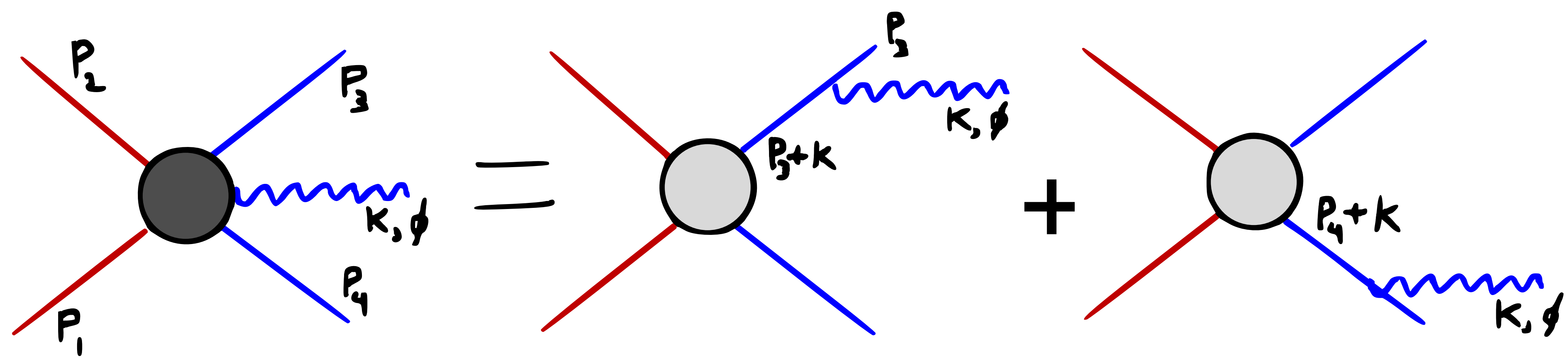
$$A_4 \cdot \frac{1}{(p_3+k)^2+i\epsilon} \cdot s(\{k,\phi\},p_3) + A_4 \cdot \frac{1}{(p_4+k)^2+i\epsilon} \cdot s(\{k,\phi\},p_4)$$

$$s(\{k,h\},p_i) = f(k.p_i)\tilde{J}_h(\rho z_i)$$

Next-simplest case: $f = \frac{q_i}{\mu} p_i.k$

high-energy growth of f cancels propagator suppression

More general interactions?

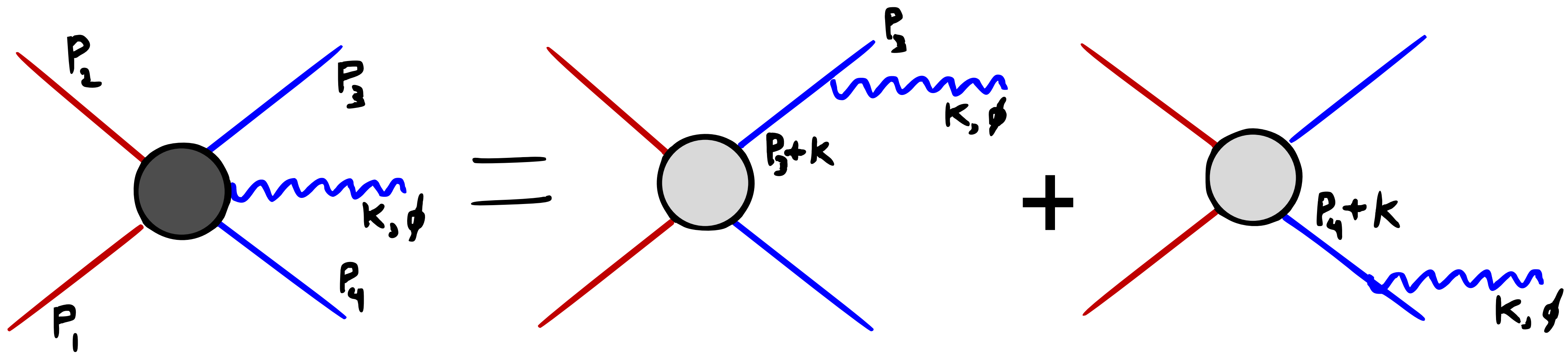


$$A(12 \to 34\{k,h\}) = A_4 \left[\frac{\overbrace{p_3.k \; q_3/\mu}^{f \text{ factor}}}{2p_3.k + i\epsilon} \tilde{J}_n(\rho z_3) + (3 \leftrightarrow 4) \right]$$

$$A_{h=0} \approx \frac{A_4}{2\mu} \left[(q_3 + q_4) + \mathcal{O}(\rho z)^2 \right]$$

Leading term violates perturbative unitarity at energies >μ – a UV cutoff

More general interactions?



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f factor

$$A_{h=0} \approx \frac{A_4}{2\mu} \left[(q_3 + q_4) + \mathcal{O}(\rho z)^2 \right]$$

Leading term violates perturbative unitarity at energies >μ – a UV cutoff

...unless $q_3+q_4=0$ (q is conserved "charge")

Gauge Correspondence

$$s(\{k,h\},p_i) = q_i \frac{p_i.k}{\rho} \tilde{J}_h(\rho z_i)$$

If q_i is conserved in all interactions, the high-energy growth cancels in sum over all legs.

$$s_{h=0} = \text{cancel} + q_i \epsilon_+^* .p_i \mathcal{O}(\rho z)$$

$$s_{h=1} = q_i \epsilon_+^* .p_i (1 - \mathcal{O}(\rho z)^2)$$

$$s_{h=2} = q_i \epsilon_+^* .p_i \, \mathcal{O}(\rho z)$$ etc.

*Charge conservation from **perturbative unitarity** implies $h=\pm1$ dominance*

Gravity Correspondence

$$s(\{k,h\},p_i) = \frac{1}{M_P}\left(\frac{p_i.k^2}{\rho^2} + p_i^2/4\right)\tilde{J}_h(\rho z_i)$$

Similarly, quadratic term naively $(p_i.k)^2/\Lambda^3$ but equivalence principle tames high-energy growth of h=0 and h=1 interactions

$\Rightarrow$ h=2 dominates* for $\rho<E<M_P$ (with graviton-like amplitude) and cutoff delayed to Λ^3/ρ^2

* gravitational-strength h=0 couplings also generated by simplest quadratic f, but not required

Helicity Correspondence Summary

Lorentz invariance and unitarity allow simple (but highly constrained) amplitudes:

$$s(\{k,h\},p_i) = f(k.p_i)\tilde{J}_h(\rho z_i)$$

– For generic f, h=0 interaction dominates at $E \gg \rho$

– Constrained cases where h=1 (2) dominate

Charge conservation/equivalence principle from *perturbative unitarity*

Approximated by usual helicity amplitudes

– No correspondence above h=2

Higher powers of $p.k$ are like higher-derivative couplings; h>2 never dominates

Thermodynamics

Infinite no. of polarizations $\Rightarrow$ infinite vacuum heat capacity[Wigner '62]

Does coupling to CSPs make a system supercool?

- Do all CSP states reach thermal equil.?
- What about low-energy phase-space, $E \sim \rho$?

Correspondence suggests both can be avoided

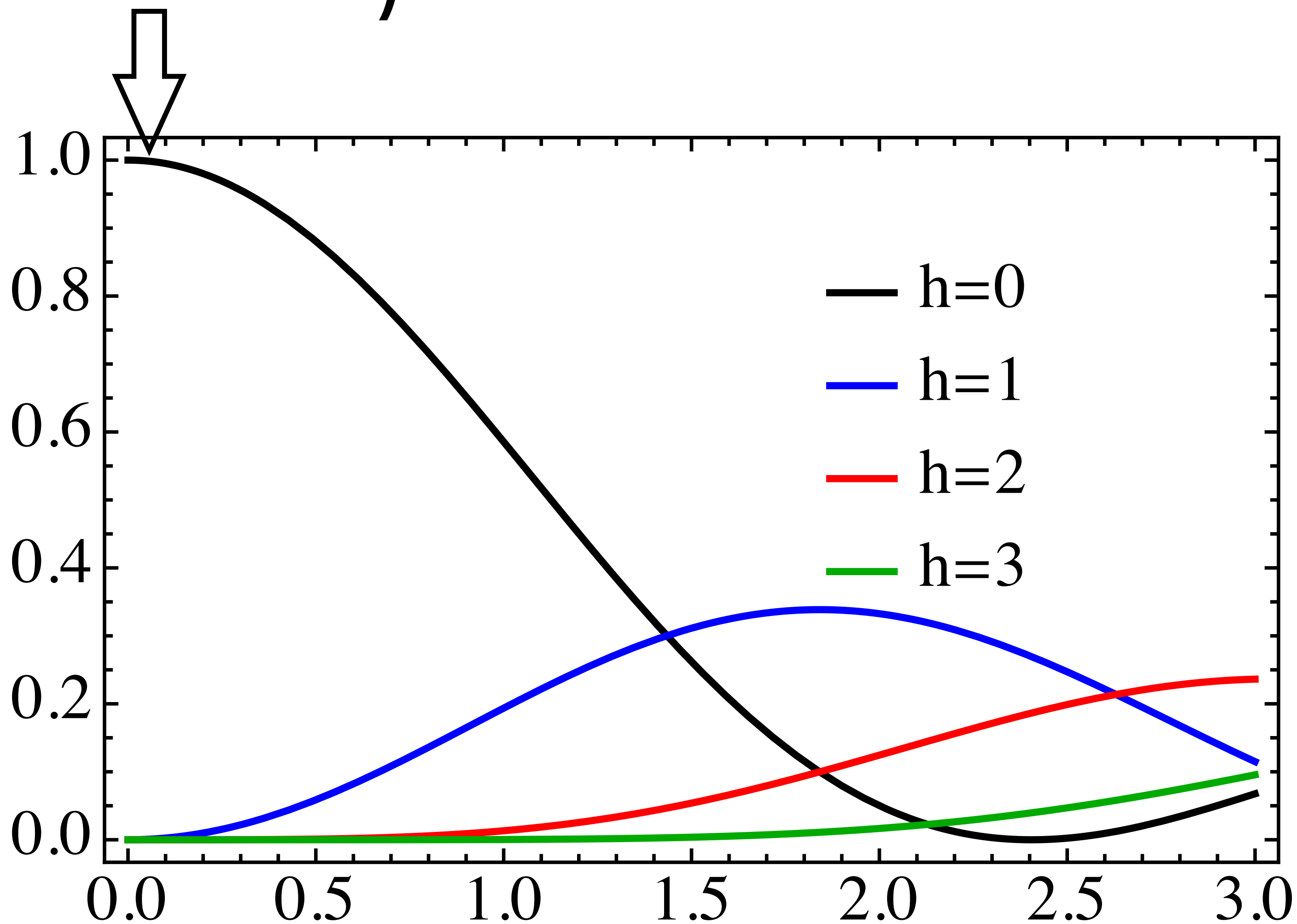
(correspondence beyond soft factors is only a conjecture, but plausibly protected by unitarity)

Thermodynamics in a Nutshell

$E{\sim}T{\gg}\rho \;\Rightarrow$ small z

$$\sigma_h \propto |s(z,h)|^2$$

$$\sim \sigma_0 (\rho v_{th}/T)^{2h}/h!^2$$



$\rho z_i \sim \rho/E$

h=0 has microscopic thermalization time τ_0,

For h≠0, $\tau_h \sim \tau_0 (T/\rho v_{th})^{2h}/h!^2 \gg \tau_0$

Long-lived thermal systems ⇒ bound on ρ

□

Thermodynamics: Early Universe

If photon is helicity-1 part of a CSP with gauge correspondence, how small must its ρ be?

$$\tau_h \sim \tau_0 (T/\rho v_{th})^{2h} / h!^2$$

h≠1 production dominated by Compton at T~MeV

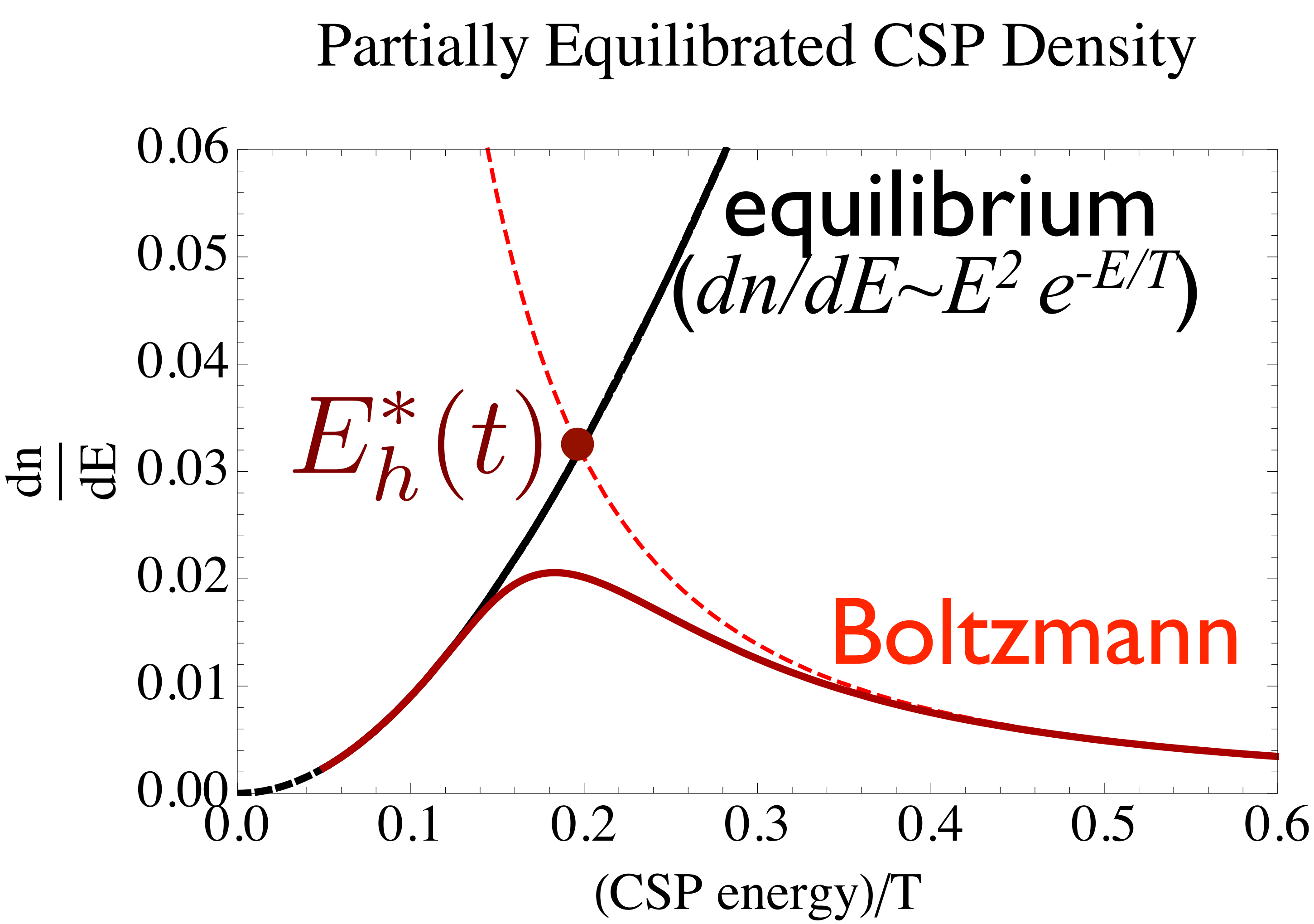
$$\tau_{h=0,2} \sim \tau_\gamma (T/\rho)^2 \gg H^{-1}(T)$$

⇒ For ρ ≲ meV, CSP partner polarizations don't thermalize

Thermodynamics: Closer Look

Phase-space density of *h*'th CSP mode at time *t* :

$$\frac{d\dot{n}_h}{dE} = \overbrace{n_e^2 \langle \frac{\sigma_{Brem} v}{dE} \rangle}^{E^2/\tau(E)} J_h\left(\frac{\rho v}{E}\right)^2 \left(1 - \frac{dn_h/dE}{dn/dE_{eq}}\right)$$



$$n_h(t) \sim E_h^*(t)^3$$

$$\rho_h(t) \sim E_h^*(t)^4$$

First two factors dictate scaling

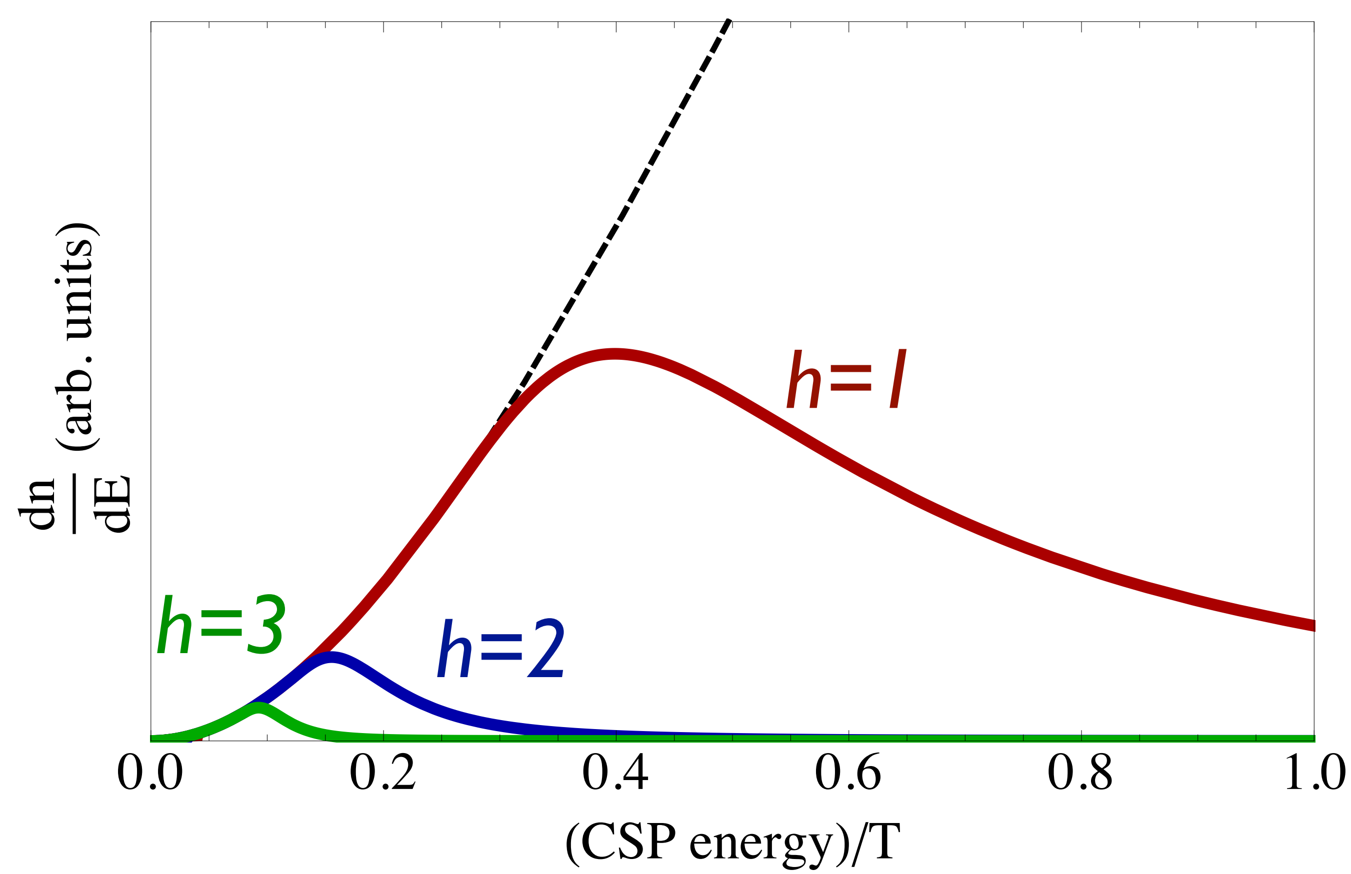
$$\frac{t}{\tau(E^*)} J_h \left(\frac{\rho}{E_h^*}\right)^2 = 1 \Rightarrow E_h^* \sim \rho \left(\frac{t}{h!^2 \tau}\right)^{1/2h}$$

$E_{h=1}^* \ll T$ is old non-thermalization condition

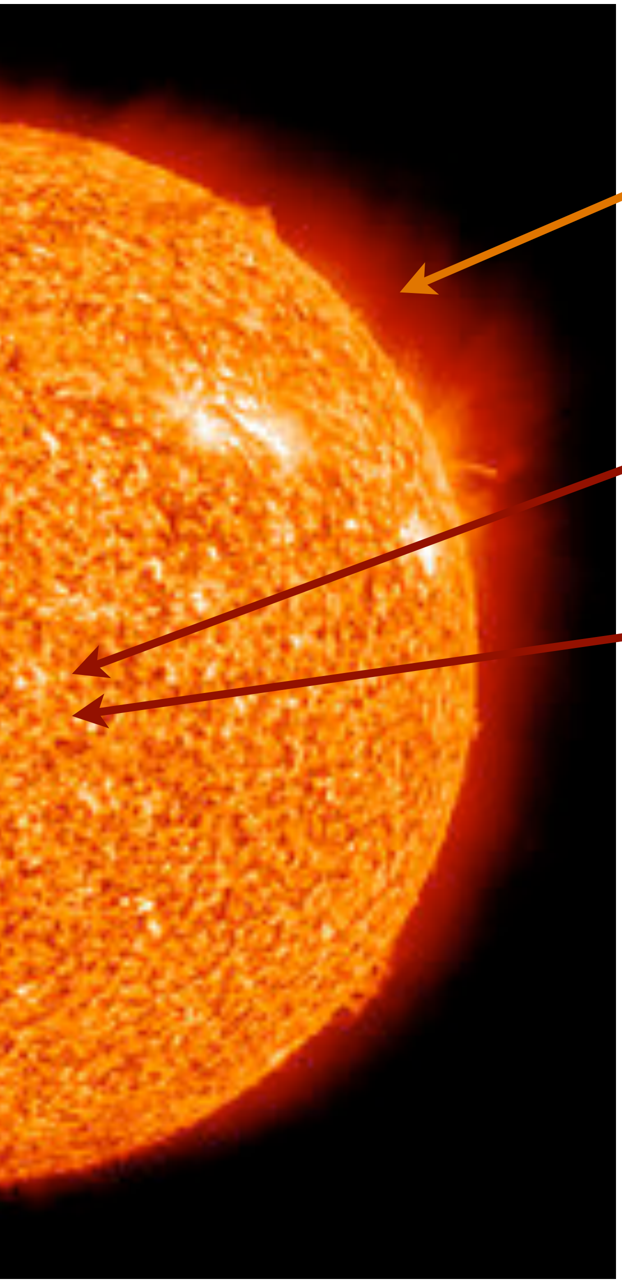
E^* decreases with h
and → ρ/h at large h

Total entropy (energy) density in all high-h CSPs
$\sim \sum (\rho/h)^{3(4)}$
highly convergent

Partially Equilibrated CSP Density



Solar Cooling Constraint on CSP Photon



Luminosity ~ 10^{34} erg/s

T ~ 10^{7} K, density ~ 5 g/cm^{3}

Power$_{(brem)}$ ~ 10^{59} erg/s

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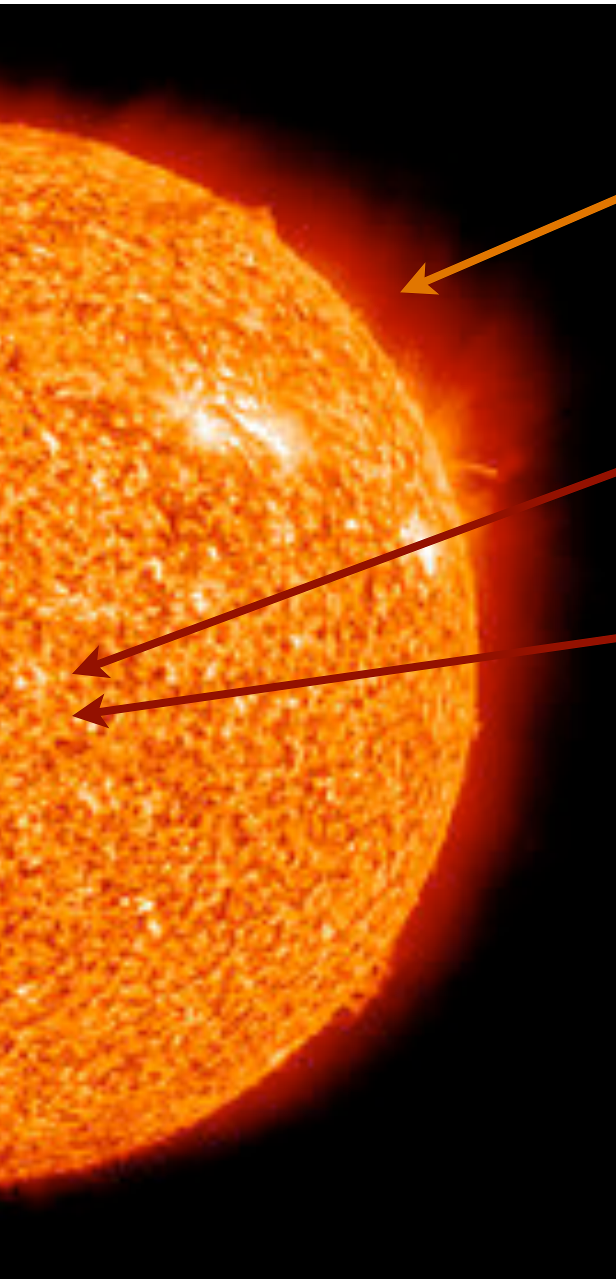

Luminosity ~ 10^{34} erg/s

T ~ 10^7 K, density ~ 5 g/cm^3

Power$_{\text{(brem)}}$ ~ 10^{59} erg/s

If *one* h≠1 CSP brem'd per 10^{26} γ's , luminosity and stellar evolution would change by O(0.1).

$$\rho^2 \lesssim 10^{-26} m_e T \sim (10^{-8}\text{eV})^2$$

$$\rho^{-1} \gtrsim 10\text{m}$$

Lower-energy CSPs and cooler stars ⇒ few-10x stronger bound on ρ

CSP Thermodynamics: Bottom Line

Helicity correspondence of amplitudes $\Rightarrow$

- Helicity-like physics for $E \gg \rho v$
- Viable *approximate* thermodynamics

Thermodynamic corrections from ρ≠0 are

- calculable
- dominated by **one** nearest-neighbor helicity

e.g. for CSP photon:

- early-universe $\delta g_* \ll 1$ if $\rho \lesssim 10^{-4}$ eV
- tightest known constraint: stellar cooling $\Rightarrow$ $\rho \lesssim 10^{-9}$ eV.

Summary – CSP Amplitudes

- Theory
  - Soft factor limits exist (unlike high helicity)
  - Tree level CSP scattering amplitudes with appropriate factorization limits exist
  - Perturbative unitarity $\Rightarrow$ any CSP theory will be approximated by a gauge theory with h=0,1,2 in the $\rho \rightarrow 0$ limit (helicity correspondence)
- Phenomenology
  - correspondence $\Rightarrow$ known gauge theories may be degenerate limits of CSP theories
  - *calculable* approximate thermodynamics
  - tests in classical limit are important – *presently limited by theoretical control, but may be testable soon*

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Spacetime interpretation of CSPs?

Try to pick up where S-matrix arguments left off

- Multi-emission, CSP exchange
- Classical limit
- Unfamiliar phase structure in soft factors
  where does it come from?
  is it local enough to guarantee causality?

Aim for manifest helicity correspondence

Connect to tensor-field e.o.m. for gauge th'y
All spins on same footing (in free theory)

Fronsdal Formalism

Consider a "polynomial" field in an auxiliary spin-space

$$\psi(x,\omega) = \phi(x) + \omega^\mu A_\mu(x) + \frac{1}{2}\omega^\mu\omega^\nu h_{\mu\nu} + \frac{1}{3!}\omega^\mu\omega^\nu\omega^\rho G_{\mu\nu\rho}\ldots$$

Equation of motion for components:

$$-\Box_x\phi - J = 0$$

$$\omega^\mu\left(\Box_x A_\mu - \partial_\mu\partial\cdot A - J_\mu\right) = 0$$

$$\omega^\mu\omega^\nu\left(\Box_x h_{\mu\nu} + \cdots - \bar{J}_{\mu\nu}\right) = 0$$

etc…

Unifying structure?

Fronsdal Equation

Consider a "polynomial" field in an auxiliary spin-space

$$\psi(x,\omega) = \phi(x) + \omega^\mu A_\mu(x) + \frac{1}{2}\omega^\mu\omega^\nu h_{\mu\nu} + \frac{1}{3!}\omega^\mu\omega^\nu\omega^\rho G_{\mu\nu\rho}\ldots$$

Double-traceless condition $\Box_\omega^2\psi = 0$

Fronsdal eom:

$$\left(-\Box_x + \omega\cdot\partial_x\partial_\omega\cdot\partial_x - \frac{1}{2}(\omega\cdot\partial_x)^2\Box_\omega\right)\psi(\omega,x) = J(\omega,x)$$

Gauge invariance: $\delta\psi = i\omega\cdot\partial_x\epsilon$ with $\Box_\omega\epsilon = 0$

Trace conditions ⇒ right d.o.f. at ranks ≥3

Fronsdal → CSPs?

At least two generalizations of Fronsdal equations contain CSPs:

Common ingredients:

1. Deformed gauge redundancy $\delta\psi = (i\omega\cdot\partial_x + \rho)\epsilon$

2. Deform trace conditions $(\partial_\omega^2 - 1)^2\psi = 0$
(cf Bekaert and Mourad '06)

→ one CSP

OR

Generalize away from polynomial ψ:
(Schuster and NT, arXiv:1302.3225)

$$\psi(x,\omega) \neq \phi(x) + \omega^\mu A_\mu(x) + \frac{1}{2}\omega^\mu\omega^\nu h_{\mu\nu} + \frac{1}{3!}\omega^\mu\omega^\nu\omega^\rho G_{\mu\nu\rho}\ldots$$

→ CSPs with all ρ

CSP Covariant Equation of Motion

[PS and Toro; Bekaert and Mourad]

$$\psi(x,\omega) = \phi(x) + \omega^\mu A_\mu(x) + \frac{1}{2}\omega^\mu\omega^\nu h_{\mu\nu} + \frac{1}{3!}\omega^\mu\omega^\nu\omega^\rho G_{\mu\nu\rho}...$$

Double-traceless condition $(\partial_\omega^2 - 1)^2\psi = 0$

$$\left(-\Box_x + (\omega\cdot\partial_x + \rho)\partial_\omega\cdot\partial_x - \frac{1}{2}(\omega\cdot\partial_x+\rho)^2(\Box_\omega - 1)\right)\psi(\omega,x)$$

$$= J(\omega,x)$$

Gauge invariance: $\delta\psi = (\omega\cdot\partial_x + \rho)\epsilon$

with $(\partial_\omega^2 - 1)\epsilon = 0$

The Need for Deformed Gauge Redundancy:

Helicity +*h* wavefunction,

$$\psi_h = \omega_{\mu_1} \ldots \omega_{\mu_h} \epsilon_+^{\mu_1} \ldots \epsilon_+^{\mu_h}$$

"*Lowering*" LG generator $T_- = -\omega.k\epsilon_-.\partial_\omega + \omega.\epsilon_-k.\partial_\omega$

$$T_-\psi_h \propto \omega_{\mu_1} \ldots \omega_{\mu_h} \epsilon_+^{\mu_1} \ldots \epsilon_+^{\mu_{h-1}} k^{\mu_h}$$

Usual redundancy $\delta\psi = i\omega \cdot \partial_x \epsilon$ ensures $T_-\psi_h \simeq 0$

CSP redundancy $\delta\psi = (i\omega \cdot \partial_x + \rho)\epsilon$
allows $T_-\psi_h \simeq \rho\psi_{h-1}$

Deformed trace condition/non-polynomial branch?

No finite-rank tensor transforms as CSP state

"Lowering" LG generator $T_- = -\omega.k\epsilon_-.\partial_\omega + \omega.\epsilon_- k.\partial_\omega$

$(T_-)^m$ annihilates all tensors of rank <m/2

but never annihilates CSP state, just lowers:

$$(T_-)^m \psi_h \simeq \psi_{h-m}$$

$\Rightarrow$ CSP wavefunctions have

infinite tower of non-zero tensor components
or non-tensor dependence on ω

Relaxing polynomial restriction, it is natural to interpret double-trace condition as localization to null cone in Fourier-conjugate space:

$$\tilde{\psi}(\eta, x) \equiv \int d^4\omega e^{-i\eta\cdot\omega} \psi(\omega, x)$$

$$(\partial_\omega^2)^2 \psi(\omega) = 0 \longleftrightarrow (\eta^2)^2 \tilde{\psi}(\eta) = 0$$

Define $\tilde{\psi}$ in terms of *unconstrained* field:

$$\tilde{\psi}(\eta, x) = \delta'(\eta^2) \psi(\eta, x)$$

(similarly for gauge parameter)

In terms of the unconstrained field $\psi(\eta, x)$:

eom: $-\delta'(\eta^2)\Box_x\psi + \frac{1}{2}\Delta\left(\delta(\eta^2)\Delta\psi\right) = \delta'(\eta^2)J$

gauge variation: $\delta\psi = \left(\eta \cdot \partial_x - \frac{1}{2}\eta^2\Delta\right)\epsilon(\eta, x)$

where $\Delta = \partial_\eta . \partial_x + \kappa$

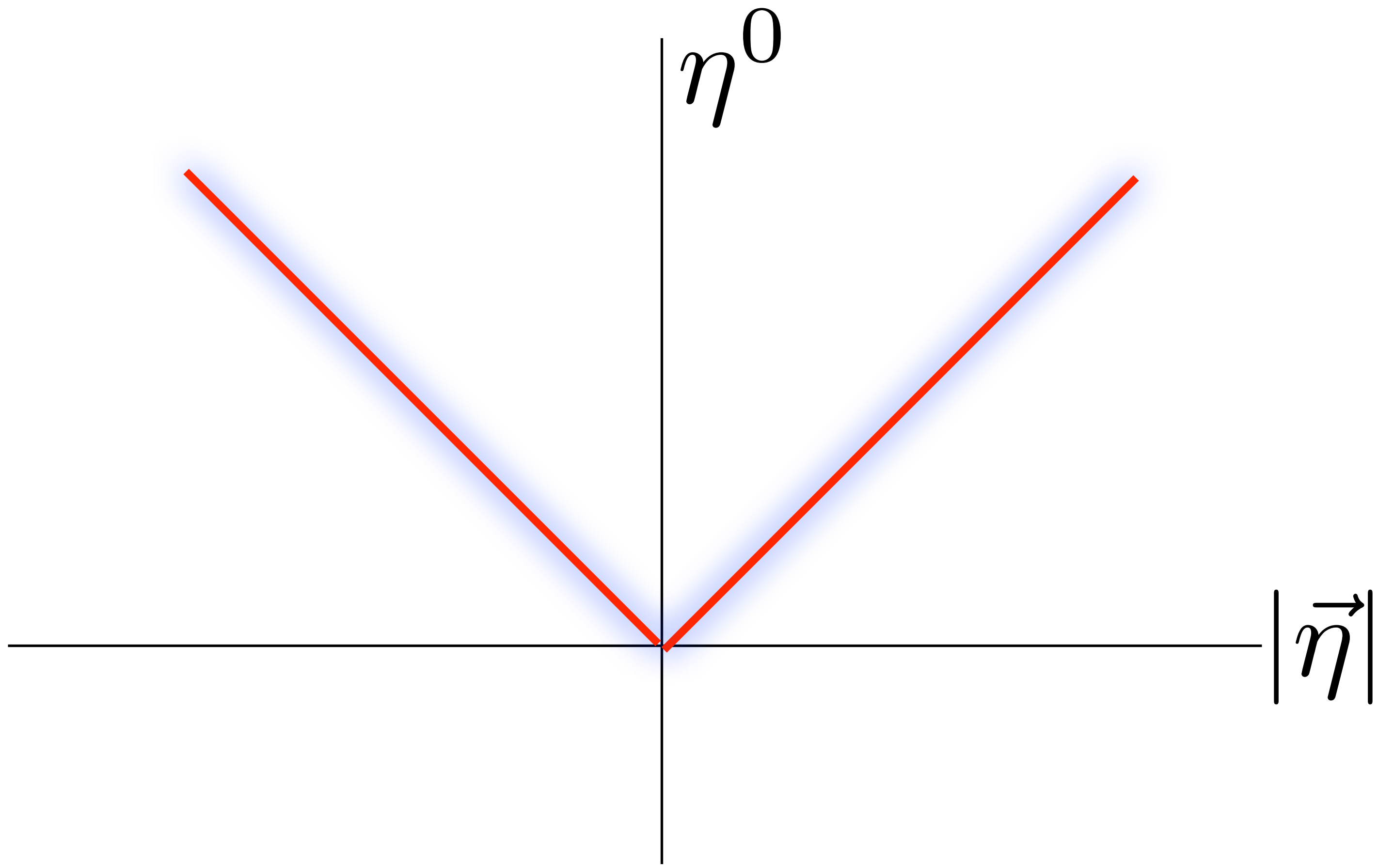
This eom is the variation of a quadratic, local, gauge-invariant action that propagates CSPs of ***all*** ρ

$$S = \int d^4x d^4\eta \left[\delta'(\eta^2)(\partial_x\psi)^2 + \frac{1}{2}\delta(\eta^2)(\Delta\psi)^2\right] + \delta'(\eta^2)J\psi$$

[Schuster & NT 1302.3225]

Physical Degrees of Freedom

Component Decomposition of ψ near null cone:



$$\psi(\eta, x) = A(\vec{\eta}, x) + \frac{\eta^0}{|\vec{\eta}|} B(\vec{\eta}, x) + O((\eta^2)^2)$$

non-physical

Physical Degrees of Freedom

Component Decomposition of ψ near null cone:

$$\psi(\eta, x) = A(\vec{\eta}, x) + \frac{\eta^0}{|\vec{\eta}|} B(\vec{\eta}, x) + O((\eta^2)^2)$$

dynamical   non-dynamical   non-physical

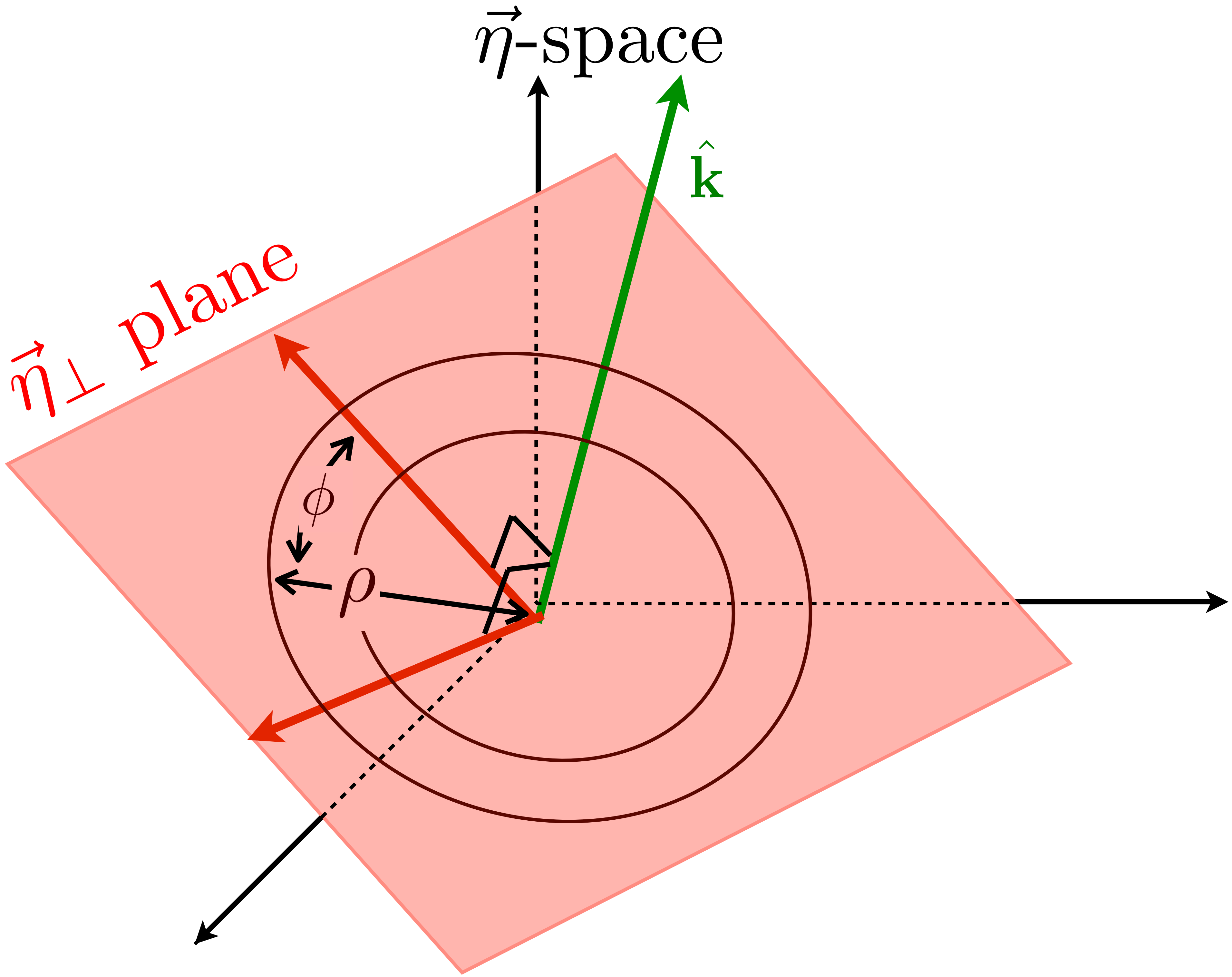
Residual gauge freedom fixed by Coulomb-like condition

$$(-\vec{\nabla}_x . \vec{\nabla}_\eta + \kappa)A = 0$$

Straightforward canonical quantization (like Coulomb-gauge QED) for background J

$\Rightarrow$ Physical d.o.f live on (D-2)-dimensional $\vec{\eta}_\perp$ plane

Physical Degrees of Freedom



$\vec{\eta}_\perp$ plane ***is*** Little-Group “momentum” space

Summary and Questions

Covariant field models of one or many CSPs

Gauge redundancy is crucial to consistency! (explains failure of previous field theory constructions)

Smooth $\rho \to 0$ limit

Open questions

- Covariant ***action*** for one-CSP theory?
  eom & gauge-fixed Hamiltonian exist
- Appropriately conserved ***matter currents***?
  connection to soft factors is a guide
- Are there ***local*** **G-I operators**?
- Coupling to gravity?

Rapid progress towards a physically clear theory with sharp predictions

Conclusions – Making Sense of CSPs

- Phenomenology
  - correspondence ⇒ CSPs more consistent than they appear at first glance
  - *calculable* approximate thermodynamics
  - tests in classical limit are important – *presently limited by theoretical control, but may be testable soon*

- Theory
  - want spacetime interpretation for CSPs, *interactions with matter and gravity*
  - found gauge field theories coupled to background currents; many more questions
  - worldline or extended object pictures?

Thanks!

Backup

Little Group Generators:

$$\mathbf{T}_{1,2} \equiv \vec{\epsilon}_{1,2}.(\vec{\mathbf{K}} \times \vec{k} + \vec{\mathbf{J}}k^0) \qquad \mathbf{R} = \vec{\mathbf{J}}.\hat{k}$$

$$\mathbf{T}_\pm \equiv \mathbf{T}_1 \pm i\mathbf{T}_2$$

$[\mathbf{R}, \mathbf{T}_\pm] = \pm T_\pm$ $\Rightarrow$ raising and lowering

$$T_\pm |k, h\rangle = \rho_{\pm,h} |k, h \pm 1\rangle$$

unitarity $\Rightarrow$ $\rho_{+h} = \rho^*_{-,h+1}$

$[\mathbf{T}_+, \mathbf{T}_-] = 0$ $\Rightarrow$ $|\rho_{+h}|^2 = |\rho_{+,h+1}|^2$

Remove phases by choice of basis

[back]

CSP Soft Factors and Unitarity

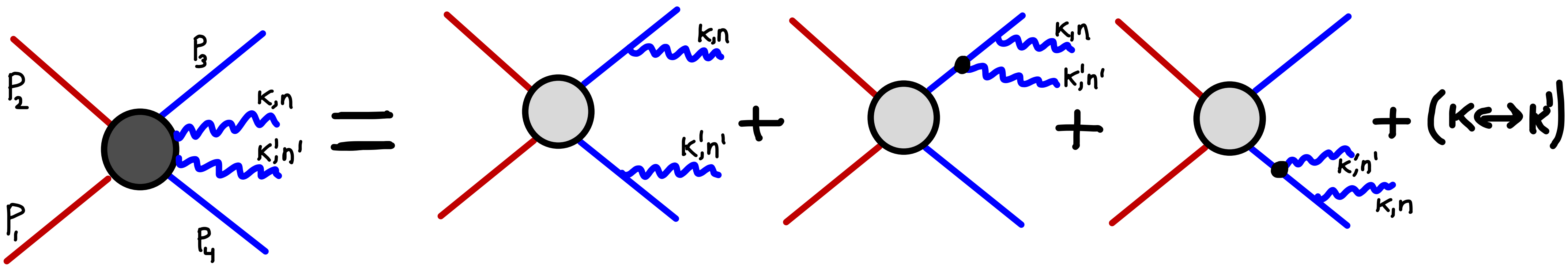
$$s(\{k,n\},p_i) = \tilde{J}_n(\rho z_i) = \left(\frac{\epsilon_+.p}{k.p}\right)^n \sum_{j=0}^{\infty} c_j \left(\frac{\rho^2 \epsilon_+.p\,\epsilon_-.p}{(k.p)^2}\right)^j$$

Almost-everywhere analytic in p, k, $\varepsilon_\pm$ (power series of J_n) with isolated essential singularity at $z \to \infty$ (i.e. k soft or collinear)

- *Bounded* (by 1) *for all real momenta*
  $\Rightarrow$ no iε deformation (unlike multi-pole)
- No spurious imaginary part in optical th'm

Also demand existence of multi-particle amplitudes with consistent factorization limits...

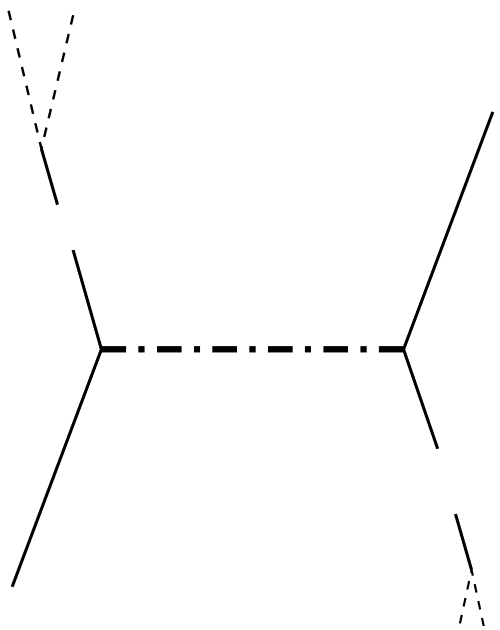
Multi-CSP Amplitudes



Using soft factor (f=const) as a sewing rule yields candidate two-CSP amplitudes (and beyond) that factorize appropriately and maintains scalar-correspondence [PS & Toro 1302.1577]

For gauge- and gravity-correspondence, don't know general sewing rules yet (expect them to be more complex)

Unitarity of CSP-Exchange Amplitudes



Candidate

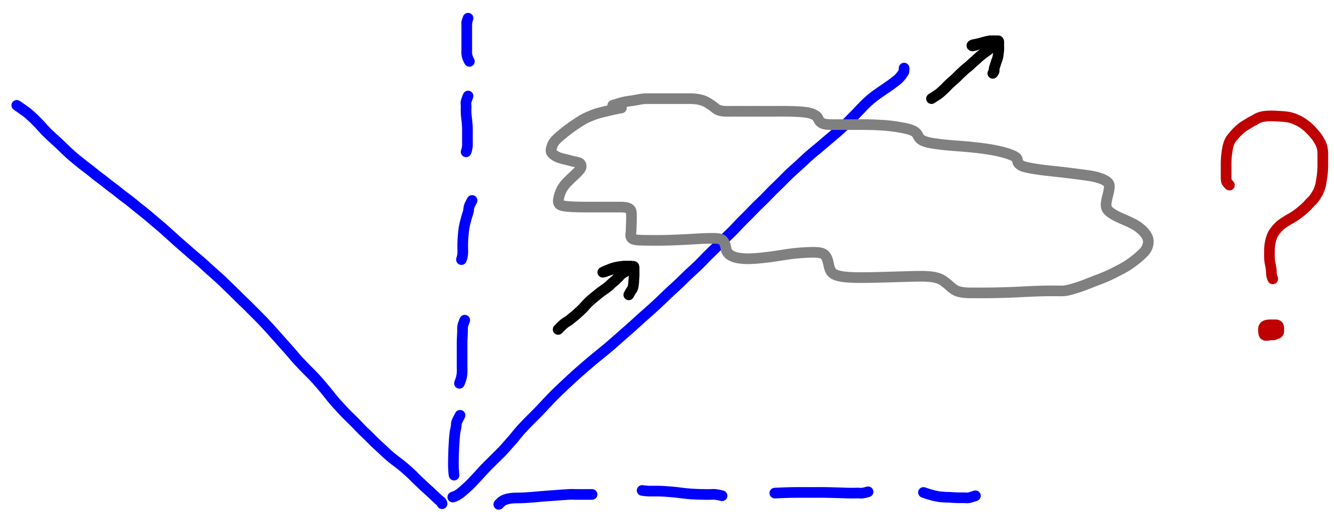
$$\mathcal{M}_4 = \frac{1}{k^2 + i\epsilon} J_0\left(\frac{\rho\sqrt{-(\epsilon^{\mu\nu\rho\sigma} k_\nu p_\rho q_\sigma)^2}}{k.p\, k.q + \alpha p.q k^2 + \ldots}\right)$$

$k^2 \longrightarrow 0$ limit fixed by unitarity;
ambiguity in $O(k^2)$ corrections

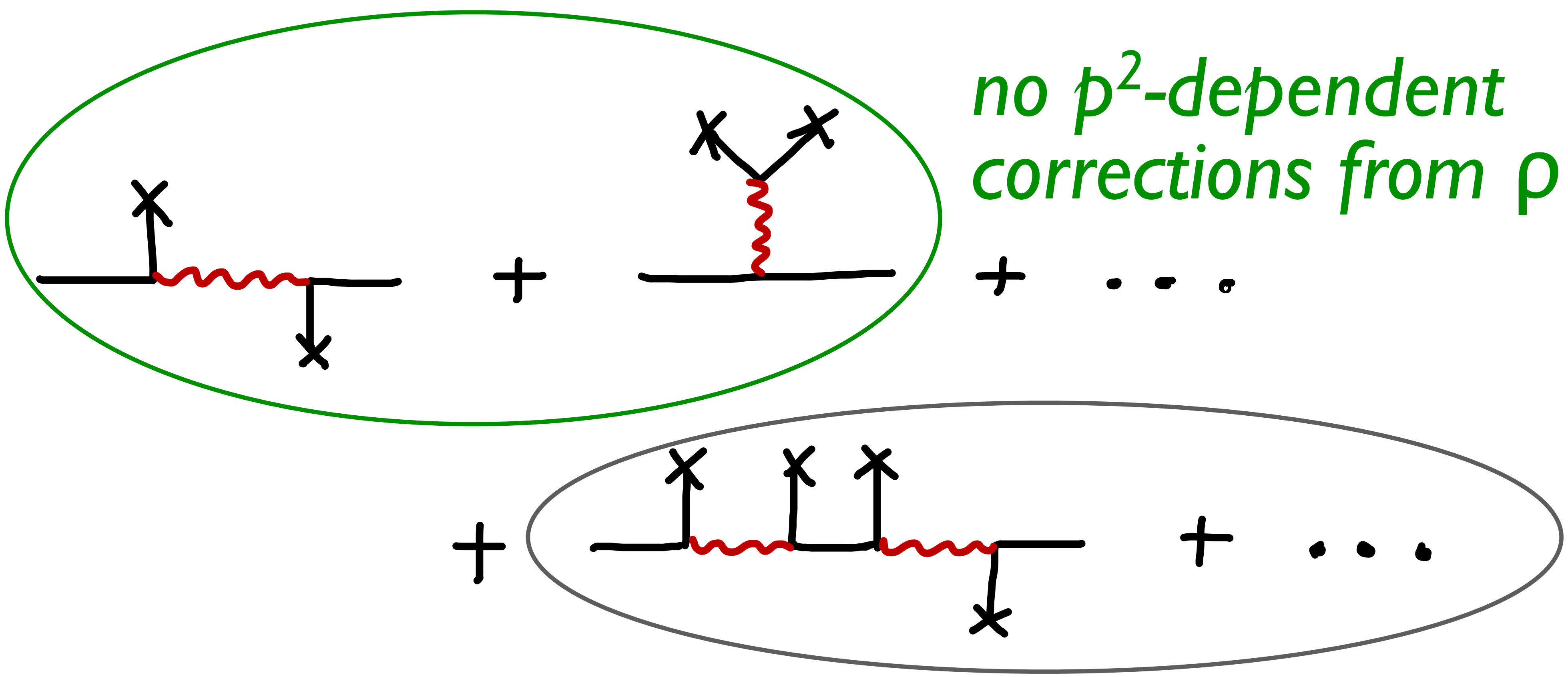
Correspondence limit: $$M_4 \sim \frac{1}{k^2 + i\epsilon}\left(1 - \mathcal{O}\left(\frac{\rho|\mathbf{v} \times \mathbf{k}|}{k^2}\right)^2\right)$$

Causality & Analyticity

Matter propagation through a background?



$$\Pi(p^2) =$$



no p^2-dependent corrections from $\rho \neq 0$

work in progress

Matrix elements with "Stress Energy" Tensor

In contrast to Weinberg-Witten argument forbidding high-helicity matrix elements with a covariant stress-energy tensor

$$\langle p', \phi' | T^{\mu\nu}(k) | p, \phi \rangle = (p^\mu p'^\nu + p'^\mu p^\nu - p.p' g^{\mu\nu}) e^{i\rho\left(\frac{\epsilon_{\phi'}(p').k}{p'.k} - \frac{\epsilon_\phi(p).k}{p.k}\right)}$$

Continue to exhibit helicity correspondence – no thermo. problem…physically odd (single-exchange fwd. scattering mixes states maximally)

Coupling CSP action to helicity-2 gravity could be informative!

Don't forget about graviton-correspondence CSP

Spinor Helicity analogue
(corresponds to "*q*-lightcone gauge" ϵ)

$$p.\sigma = \lambda^{\alpha}\bar{\lambda}^{\dot{\alpha}} \qquad \epsilon_{+}.\sigma \propto \frac{\mu^{\alpha}\bar{\lambda}^{\dot{\alpha}}}{\mu^{\alpha}\lambda_{\alpha}}$$

$$\phi = \frac{\langle \lambda \mu \rangle}{[\bar{\mu}\bar{\lambda}]}$$

Wavefunction

$$\psi(\lambda^{\alpha},\bar{\lambda}^{\dot{\alpha}},\xi^{\alpha},\bar{\xi}^{\dot{\alpha}}) = f(\langle \xi \lambda \rangle, [\bar{\xi}\bar{\lambda}])e^{i\rho\left(\frac{\langle \xi \mu \rangle}{\langle \xi \lambda \rangle} + \frac{[\bar{\xi}\bar{\mu}]}{[\bar{\xi}\bar{\lambda}]}\right)}$$

Soft factor

$$s(\lambda^{\alpha}, p_{*}) = \psi(\lambda, \xi)\Big|_{\xi^{\alpha} = p.\bar{\sigma}\bar{\lambda}_{\dot{\alpha}}}$$